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Scalar and Vector Risk in the General Framework of Portfolio Theory — A Convex Analysis Approach

by

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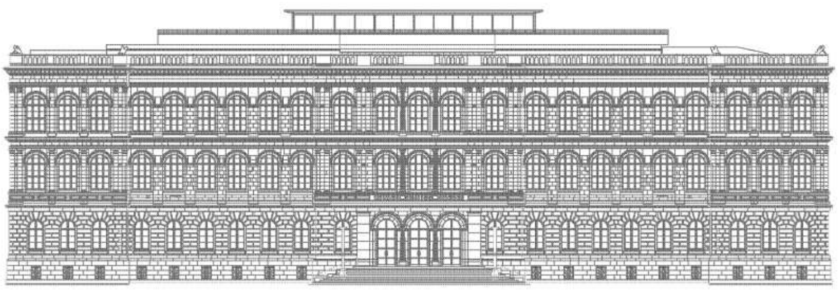

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Preface

This monograph is a summary of our recent work on portfolio theory and its applications to bank balance sheet management problems. The book consists of three main blocks: We start with a concise exposition of our recent work on a general framework for portfolio theory extending the framework of Markowitz portfolio theory to involve more general concave utility functions and convex risk measures. This framework encompasses both Markowitz portfolio theory and the growth optimal portfolio theory as special case. Next, we discuss our new results on portfolio theory allowing multiple types of different risks. This line of investigation is motivated by practical financial applications. Bank balance sheet management problems are typical examples. A bank balance sheet usually involves interest risk, credit risk, liquidity risk and other types of risks. In the course of their activity, banks and regulators prescribe limits on the different risks. Also, some risks are not comparable and thus become very difficult to aggregate in practice (e.g. funding liquidity risk and credit risk). Hence, following the idea of Markowitz to trade–off among rewards and risks, one needs to deal with a multi–objective optimization problem.

We analyze the structure of the Pareto efficient frontier for such a trade–off and its representations and, thus, provide a theoretical foundation for implementing trade–off between reward and risks in practical problems. Moreover, we illustrate the application of our theory using bank balance sheet management problems of different levels of complexity. Our emphasis here is the general pattern. We begin considering the simple expected return and discuss a linear risk model as well as linear–quadratic risk models with explicit solutions. The role of convex duality is emphasized. Furthermore, we lay out the financial meanings of both primal and dual solutions including conditions characterizing Pareto efficient bank balance sheets. These results are useful in guiding the practitioners in their decision making.

We also introduce several practically relevant risk functions, for instance, the tracking error and the logarithmic drawdown, as well as utility functions such as the logarithmic terminal wealth relative. Further, we discuss several applied portfolio optimization problems arising from various combinations of these risk and utility functions, in particular, the qualitative structure of their efficient frontier.

This monograph unifies and extends several earlier approaches of the authors in the field of portfolio analysis. It is intended for graduate students and researchers in the area of financial mathematics and applied mathematics, as well as practitioners. To make it accessible and useful to the potential readers, we carefully lay out the details for our main theory and illustrate them with applications in bank balance sheet management models that are useful for practitioners.

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Chapter 1
Introduction

This monograph gives a concise exposition of our recent work on a general framework for portfolio theory with applications. In particular, it contains our new results on portfolio problems involving multiple types of risks motivated by bank balance sheet management problems. Applications to the bank balance sheet problems, in different levels of detail, are discussed as illustrative examples.

The foundation for modern portfolio theory was established in the pioneering work of Markowitz [36, 37]. The key idea is to view portfolio selection as a trade–off between the expected return of the portfolio and its risk measured by the variance. This trade–off happens on the so–called Markowitz bullet, which is essentially the Pareto efficient frontier in the space of risk and reward. In the setting of Markowitz portfolio theory, the efficient portfolios corresponding to this efficient frontier have an affine linear representation, which makes the trade–off easy to implement. The textbook Elton et al. [19] contains an excellent discussion on implementation issues of this theory in investment practice.

A different school of thought, called growth optimal portfolio theory, is based on maximizing the utility of a portfolio as proposed by Kelly [29] and Lintner [30]. This approach also gained much attention from practitioners and was used in diverse areas, from playing Blackjack, dealing with stock portfolios to derivative trading [32, 48].

Early discussions of portfolio theory assumed a one–period financial market model. Fernholz's 2002 book [20] established a foundation for dealing with markets in which the prices of assets are described by stochastic processes.

In a series of papers [34, 35], Maier–Paape and Zhu have extended the framework of Markowitz portfolio theory to involve more general concave utility functions and convex risk measures. Therein, we also emphasized the role of duality both in analyzing the efficient frontier and its corresponding financial meaning in dealing with application problems. This framework was further extended to involve discrete multi–period market models together with Platen in [33, 39]. The multi–period model provides a natural framework to conduct model calibration using historical data.

In this theory, several risk functions can be used. Important risk functions are defined by the variance as in [36, 37], the tracking error as in [18], (conditional) Value–at–Risk, see e.g. Rockafellar and Uryasev [41, 42], and the drawdown, see e.g. Chekhlov, Uryasev and Zabarankin [12], Goldberg and Mahmoud [21] and Maier–Paape and Zhu [35].

This progress sets the stage for the main focus and new contents of this monograph: portfolio theory involving multiple types of risks. The prime motivation for this line of investigation are practical financial applications. Bank balance sheet management problems are typical examples. A bank balance sheet usually involves interest risk, credit risk, liquidity risk, and other types of risks. In the course of their activity, banks and regulators prescribe limits on the different risks. Also, some risks are not comparable and thus become very difficult to aggregate in practice (e.g. funding liquidity risk and credit risk). Thus, following the idea of Markowitz to determine the efficient frontier, one needs to deal with a multi–objective optimization problem.

On the theoretical side, we put particular emphasis on the structure of the efficient frontier. In general, we show that the efficient frontier is a path–connected set over which the efficient portfolios can be represented by a continuous map. Moreover, this efficient frontier also has several natural representations. It can be represented as a subset of the graph of the concave function maximizing the utility subject to constraints of the varying risk limits. Alternatively, this efficient frontier can also be represented as a subset of the convex function minimizing one of the risk components subject to constraints of the varying risk limits for the rest of the risk components and the changing lower bound for the utility. In particular, when there is only one risk involved, the efficient frontier can be represented continuously as graphs of a convex or concave function on intervals in rewards and risk space, respectively. This makes the trade–off along the efficient frontier for scalar risk especially convenient. The structure of the efficient frontier when multiple risks are involved is more complicated. We construct an example showing that the projection of such an efficient frontier to the coordinate hyperplanes may not be convex (see Example 3.25). Also, in the vector–risk case, there may be discontinuities of the representing functions of the efficient frontier at least at boundary points (cf. Subsection 3.1.3). Thus, trading–off between different points on the efficient frontier is not trivial and may need to be dealt with using the features related to specific applications.

Bank balance sheet management problems of different levels of complexity are used to illustrate the application of the general framework of portfolio theory. Our emphasis here is the general pattern. We start considering the simple expected return and discuss a linear risk model as well as a mix of linear and quadratic risk model. These simple models have explicit solutions. More interestingly, we can explain the financial meaning of the conditions characterizing the optimal balance sheet. Duality plays an important role in the course of solving these problems. Also, the dual solution tells us about the fair price for insuring against the corresponding risk. These insights are useful for guiding the decision of bank managers. The analysis on the bank balance sheet management problems is mostly new, except for a case study published in Júdice and Zhu [28].

In the Markowitz case, all points on the Markowitz bullet correspond to optimal solutions of the underlying trade–off problem and, to efficient portfolios alike. However, in general there is a difference between optimal and efficient portfolios. For instance, on the one hand a portfolio might assume the best possible utility value and is therefore optimal with respect to the utility. On the other hand, in case there are other portfolios which assume the same utility, but have less risk, then this optimal portfolio is not efficient. We therefore put particular emphasis on the elaboration of the qualitative structure of the efficient frontier in several applied optimization problems concerning the trade–off between a scalar utility and a vector risk function.

This monograph is intended for graduate students and researchers in the area of financial mathematics and applied mathematics, as well as practitioners. The theoretical and application parts can be read separately. For the convenience of readers, a chapter–by–chapter description of the contents is included below.

Chapter 2

In Chapter 2, we start off with a state of the art summary of the general framework of portfolio theory in the scalar risk case. The main ideas here go back to Maier–Paape and Zhu [34], but we generalize that theory in many directions using also ideas from [33, 35, 39].

Accordingly, we construct efficient portfolios as trade–off between abstract extended–valued (scalar) risk and utility functions, respectively. While the risk functions are always convex, the utility functions are always concave. Section 2.1 is used to lay out the basics for such risk and utility functions on a one–period financial market, while in Section 2.2 multi–period financial markets together with the so–called fixed fraction trading ansatz come into play. With these different tools at hand not only simple risk functions like the variance can be used, but also more complex ones like (logarithmic) drawdown or maximal drawdown can be modelled. Most of these ideas essentially stem from Platen [39]. Commonly used utility functions are expected return with an auxiliary function (see Subsection 2.1.5) and the (logarithmic) terminal wealth relative (TWR) (cf. Section 2.2), which is used in growth optimal trading (cf. Vince [51], [52]).

Afterwards, in Section 2.3 the theory for the efficient frontier is explained, in particular we introduce the Pareto efficient set in the two–dimensional risk–reward space and its representations as continuous graphs over certain intervals in risk or utility space. Moreover, we here also prove new properties like the path–connectedness of the efficient frontier (cf. Corollary 2.69) which was ignored in earlier approaches, but which is essential for investors who want to adjust strategies in a continuous manner. At last, in Section 2.4, existence and uniqueness of efficient portfolios is discussed and their relation with e.g. the maximum utility optimization problem with constraint on the risk is revealed (see Problem 2.73). In fact, in the main theorem (Theorem 2.75) a continuous efficient portfolio map can be constructed which represents the efficient portfolios as a continuous graph on the efficient frontier. Section 2.4 is closed with formulas practically relevant for the calculation of the efficient frontier and with several examples of efficient frontiers occurring often in applications. Furthermore, as an application of the theory, in Example 2.81 solutions for the Markowitz setup are calculated under an additional "close to benchmark constraint", also known as tracking error bound (see [18]). This may be used to enforce diversification or, simply to urge fund managers to invest not too far from their benchmark.

Chapter 3

In contrast to Chapter 2, in Chapter 3 we allow the risk function to be vector–valued, while the utility function remains scalar–valued as before. Thus, Chapter 3 is used to develop the general framework of portfolio theory for the vector–valued risk case, which — to the best knowledge of the authors — has not yet been discussed in the literature in this generality. In contrast to Chapter 2, the corresponding maximum utility optimization problem now has a vector–valued risk constraint.

This theory extension allows as underlying financial markets one–period as well as multi–period cases alike. But like in Chapter 2, this is completely hidden in the abstract risk and utility function. The proceeding starts in Section 3.1 similar as before with a discussion of the efficient frontier, now in a space of dimension larger than two. Although the representing functions related to the efficient frontier need a more involved notation, at first glance several results seem to generalize almost canonically (cf. Proposition 3.12, Theorem 3.13 and Theorem 3.14). However, there are several points, where the theory for the vector–valued risk functions deviates. For instance, in Subsection 3.1.3 we see that the representing functions of the efficient frontier in general no longer have to be continuous. All that is left is the continuity in the interior of their domains and a partial continuity on the boundary (cf. Corollary 3.24). Also, their domains no longer need to be convex (see Example 3.25) which is in contrast to the scalar risk case where the representing functions of the efficient frontier in the two dimensional risk–reward space were defined on intervals (cf. Corollary 2.70 and Theorem 2.72). Fortunately, an important topological property like the path–connectedness of the efficient frontier could be preserved, but the effort to get that result was much more involved (see Section 3.2 and Theorem 3.43) than for scalar risk, although the known path–connectedness of the efficient frontiers for the scalar risk case could be utilized essentially in a geometrical proof. On the other hand, the efficient frontier for the vector–risk case is in general no longer closed (see examples in Subsection 3.1.3). This was different for scalar risk, where the efficient frontier is always a closed subset of the two–dimensional risk–reward space (cf. Remark 2.80).

The last subsection, Section 3.3, deals with the efficient portfolios in the vector risk case, especially the uniqueness (Theorem 3.46) in a similar setting as in Section 2.4. As a consequence, these efficient portfolios can be parameterized as various graphs, e.g. as graph of a continuous one–to–one efficient portfolio map from the efficient frontier to the portfolio space (Corollary 3.47). In addition, a link to the scalar risk case is given by defining a scalar risk function as a linear combination of the components of the vector–valued risk function. At last, we reconsider the Markowitz setup with additional close to benchmark constraint from Example 2.81, but now from a vector risk point of view. In this example expected return is the scalar utility, but besides the portfolio variance, as a second risk function, the tracking error is used. It turns out, that the qualitative structure of the efficient frontier for this seemingly simple (vector–risk, utility)–problem is already quite complicated (cf. Example 3.50, in particular Figure 3.13 for two different projections of this efficient frontier).

Chapter 4

In Chapter 4, we give several applications of the previous theory, in particular to bank asset–liability management problems. Section 4.1, describes some of the risks which corporations and banks face and motivate the need for a multi–risk framework in practice, stemming from the limits on the different risks set by banks and regulators. In the subsequent subsections, we describe some of the risk measures that banks can use, focusing on linear and quadratic measures that yield tractable solutions.

We subsequently solve different problems with increasing levels of complexity. Section 4.2 discusses the linear case, which was developed by Júdice and Zhu [28], where the authors have solved the problem using duality and showed that the dual multipliers correspond to shadow prices of credit and interest rate risk. We describe the calculation of optimal balance sheets using real–world data using this framework.

Section 4.3 tackles the linear–quadratic case, which is new. We use a quadratic interest rate risk constraint and a linear credit risk constraint in this setting.

Enforcing the non–negativity and non–positivity restrictions for assets and liabilities, respectively, makes the problem hard to solve. Furthermore, it leads to many corner solutions which are intractable to deal with in practice. Therefore, we resort again to the close to benchmark constraint that forces the solution to be in the feasible region by allowing a maximal volatility–weighted distance to a prescribed benchmark. This formulation of the problem yields higher tractability and an exact solution when the volatility–based neighborhood of the benchmark is inside the credit and interest rate risk constraints.

Moreover, the solution is much easier to implement in practice because it yields diversification of the balance sheet. Again, we apply this example to real–world data.

Section 4.4 contains several further developments. We start with two subsections concerning the general from of a linear as well as a linear–quadratic model involving more than two kind of risks. Also, we extend the discussion on the linear–quadratic model utilizing a benchmark constraint to guarantee diversification. In the last two subsections, we discuss a minimum drawdown problem with bounded variance constraint and bounded below logarithmic terminal wealth relative (log TWR) utility. In Subsection 4.4.4, we start with some rearrangements of this problem, in particular of the non–differentiable logarithmic drawdown risk function, so that numerical calculations of solutions may be done with standard interior–point algorithms. Finally, in Subsection 4.4.5, we discuss the same problem as above from a qualitative point of view. In particular, the main theorem, Theorem 4.39, finds that in this case the efficient frontier has more structure than in the general theory worked out in Section 3.1. For instance, this efficient frontier is closed and it is furthermore a simply–connected bounded two–dimensional surface (cf. Figure 4.8).

Appendix

The appendix is used to collect several well–known facts from convex analysis. In Appendix A.1 semi–continuity for extended–valued functions (on Banach spaces) is discussed. Afterwards, we provide several useful results for extended–valued convex and concave functions in Appendix A.2. Furthermore, in Appendix A.3 a standard convex programming problem is stated and solved with a Lagrange multiplier ansatz.

After setting up the basic facts for convex analysis, Subsection A.4 gives a concise description of the duality theory. The emphasis is on Lagrangian duality as it is most directly applicable to the financial problems in this book.

Chapters 1 to 4 and the Appendix A (pages 7-214) are only available in the hardcopy of the preprint.

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