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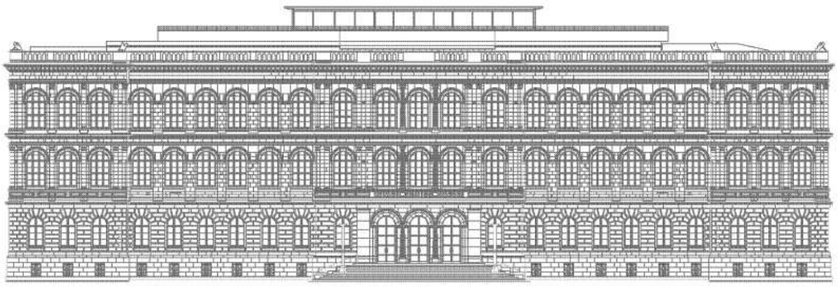

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Existence theorems for optimal fractional trading

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Abstract

In this article we show that position sizing according to a fractional trading ansatz introduced by Ralph Vince has a unique solution. This holds true for the original optimal f method as well as for the leverage space trading model with uses an additional drawdown constraint.

1 Introduction

In [4] and [5] Vince introduced the fractional trading ansatz for position sizing of portfolios. His idea was, to always invest a fixed fraction of the at the time of the investment available wealth. To determine the optimal fraction, called *optimal* f, he assumed that a historic series of N trading returns of a certain profitable investment strategy

$$t_1, t_2, \ldots, t_N \in \mathbb{R} \tag{1}$$

is available. Here t_i, $i = 1, \ldots, N$, is the profit or loss of the i–th trade. Assuming there is at least one loosing trade, we set $\hat{t} := \max\{ |t_i| : t_i < 0 \} > 0$, to be the amount of the biggest loss of that series.

$$\mathrm{HPR}_i(f) := 1 + f \cdot \frac{t_i}{\hat{t}} \geq 0\,, \quad f \in [0,1] \tag{2}$$

$$\mathrm{TWR}(f) := \prod_{i=1}^{N} \mathrm{HPR}_i(f) \tag{3}$$

are then well defined. The so called terminal wealth relative (TWR) is the ratio of the wealth obtained after these N trades when always betting with a fixed fraction $f \in [0,1]$ of the available wealth. Notice that in case $t_{i_0} = -\hat{t} < 0$ is a realization of the biggest loss, then the holding period return (HPR) of that trade is $\mathrm{HPR}_{i_0}(f) = 1 + f\,\dfrac{t_{i_0}}{\hat{t}} = 1 - f,$ yielding as worst case loss a fraction f of the available wealth.

According to Vince, the optimal fraction is that $f = f^{\mathrm{opt}}$ which maximizes the terminal wealth relative, i.e.

$$\mathrm{TWR}(f) \overset{!}{=} \max\,, \quad f \in [0,1]\ . \tag{4}$$

In all his examples it appeared that TWR(f) had a unique maximum, which he found “by looping through all values for f between .01 and 1, we can find that value for f which results in the highest TWR” (cf.[4], p. 31). Clearly that isn't a very convincing method. Our first result in section 2 therefore will show that (4) always has a unique solution $f^{\text{opt}} \in (0,1)$, if (1) is a profitable historic series, i.e. if $\sum_{i=1}^{N} t_i > 0$.

Usually the optimal f strategy of Vince results in tremendous drawdowns of the portfolio, even though the long term wealth growth is maximized. Therefore experts on position sizing rarely recommend this method (cf. e.g. van Tharp [7], Chap. 15, Model 31). In order to overcome the drawdown problem, Vince in [6] introduced a constrained optimization problem

$$\begin{aligned} \text{TWR}(f) &\overset{!}{=} \max\,, \qquad f \in [0,1]\ , \\ \text{s.t.} \qquad & RR(f,c) \le d\ , \end{aligned} \tag{5}$$

where $c, d \in (0,1)$ are given parameters and $RR(f,c)$ is the probability to exceed a drawdown of c, when investing with a fixed fraction of f. Here RR stands for risk of ruin. We will give the exact definition of $RR(f,c)$ in section 3 and furthermore show that (4) also has a unique solution. Although the new drawdown controlling ansatz (5) seems to overcome the original problems of that method, recently in [3] we showed that it is possible to extremely reduce the drawdown by using diversification of the portfolio, i.e. by parallel stochastic independent investments, where each of these investments was traded with the original optimal f fraction of its available partial wealth.

2 Optimal f lemma

In this section we will show that the original optimal f problem of Vince (4) has a unique solution, in case of a positive expectation of the historic trading series, or equivalent, if $\sum\limits_{i=1}^{N} t_i > 0$. To simplify notation, we set $a_i := t_i/\hat{t}$, $i = 1, \ldots, N$. Also, we may assume w.l.o.g. that $a_i \neq 0$ for all i, because elimination of that factor would not change the TWR.

Lemma 2.1 (optimal *f* Lemma)
Let $h(x) = \prod\limits_{i=1}^{N} \big(1 + a_i x\big)$, $x \in [0,1]$, *be a polynomial of degree* $N \geq 2$ *with* $a_i \in [-1, \infty) \setminus \{0\}$, $a_{i_0} = -1$ *for some* $i_0 \in \{1, \ldots, N\}$ *and* $\mu := \sum\limits_{i=1}^{N} a_i > 0$. *Then:*

- **(a)** $h(0) = 1$, $h'(0) > 0$, $h(1) = 0$, *and* $h(x) > 0$ *in* $[0,1)$.
- **(b)** *h has exactly one extremum x_0 in $[0,1)$. In fact x_0 is a maximum, $x_0 \in (0,1)$ and $h(x_0) > 1$ holds.*

Proof: ad (a) Everything is clear besides $h'(0) > 0$. For all $x \in \mathbb{R}$ with $h(x) > 0$ $\big($i.e. in particular all $x \in [0,1)\big)$, we have

$$h(x) = \exp\Big(\log h(x)\Big) = \exp\left(\sum_{i=1}^{N} \log\big(1 + a_i x\big)\right).$$

Therefore

$$h'(x) \;=\; h(x) \cdot \sum_{i=1}^{N} \;\frac{a_i}{1 + a_i x} \tag{6}$$

and $\;h'(0) \;=\; h(0) \cdot \sum_{i=1}^{N} a_i = \mu > 0.$

ad (b) We set $b_i := 1/a_i \in (-\infty, -1] \cup (0, \infty)$ and renumber such that

$$b_1 \;\leq\; b_2 \;\leq\; \ldots \;\leq\; b_N \quad \text{and} \quad b_{j_0} \;=\; -1\,,\; b_{j_0+1} \;>\; 0\;.$$

According to (6) we have for all $x \in \mathbb{R}$ with $h(x) > 0$

$$h'(x) \;=\; h(x) \cdot \underbrace{\sum_{i=1}^{N} \frac{1}{b_i + x}}_{=:\, g(x)}$$

Since h is positive in $[0, 1)$ we get

$$\Big(h'(x) \;=\; 0 \;\;\Longleftrightarrow\;\; g(x) \;=\; 0\Big) \;\text{ for all } x \in [0, 1)\;.$$

Therefore the discussion of extrema of h in $[0, 1)$ reduces to a discussion of zeros of g in $[0, 1)$. Using **(a)**, h has at least one extremum $x_0 \in (0, 1)$ and therefore **(b)** follows, once we can show that this is the only one. Hence it remains to show:

Claim: $\qquad g$ has at most one zero in $(0, 1)$. $\qquad$ (7)

Case 1: b_i are pairwise disjoint

In this case we can sketch the graph of g in $\mathbb{R}$ (cf. Fig. 1)
and clearly g has at least one zero between $-b_{i+1}$ and $-b_i$ for $i = 1, \ldots, N-1$, yielding at least $N-1$ zeros of g in $\mathbb{R}$. On the other hand

$$g(x) \;=\; \frac{\prod\limits_{i\neq 1} \big(b_i + x\big) \;+\; \prod\limits_{i\neq 2} \big(b_i + x\big) \;+ \cdots +\; \prod\limits_{i\neq N} \big(b_i + x\big)}{\prod\limits_{i=1}^{N} \big(b_i + x\big)}$$

with a numerator of degree $N-1$, that has at most $N-1$ zeros. As a consequence g has exactly $N-1$ zeros with exactly one in $\big(-b_{j_0+1}, 1\big) \supset (0, 1)$ yielding (7). In order to argue that g has exactly one zero

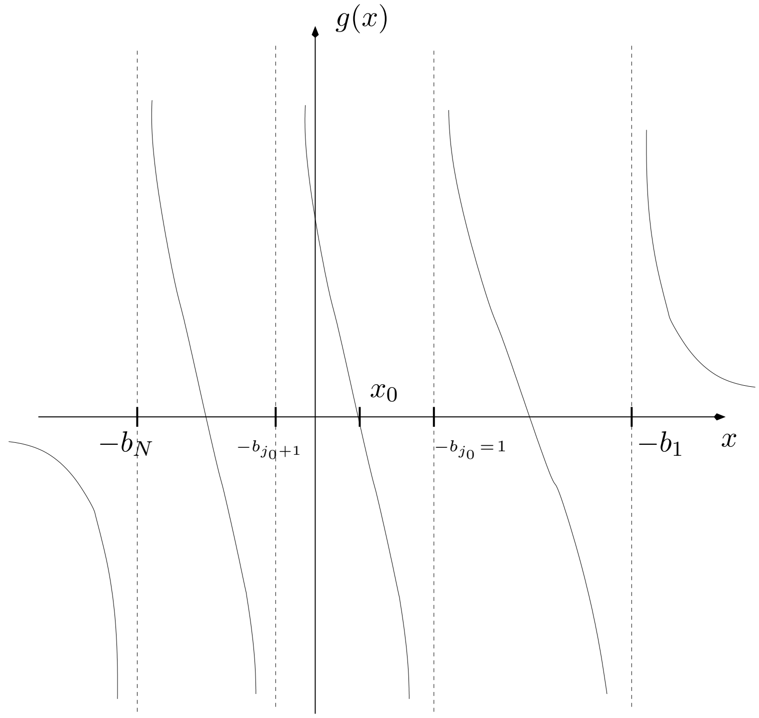

Figure 1: Zeros of g in **Case 1**, i.e. all b_i are pairwise disjoint

between $-b_{i+1}$ and $-b_i$, we could alternatively use that g is strictly monotonically decreasing (wherever it is defined) since all summands of g are monotonically decreasing (wherever they are defined).

Case 2: In case not all b_i are pairwise disjoint, we replace the b_i by $\tilde{b}_k$, $k = 1, \ldots, \tilde{N} < N$ pairwise disjoint such that $\tilde{b}_{k_0} = b_{j_0} = -1$, $\tilde{b}_{k_0+1} > 0$ and

$$g(x) = \sum_{k=1}^{\tilde{N}} \frac{\alpha_k}{\tilde{b}_k + x} \, , \, \alpha_k \in \mathbb{N} \quad \text{with} \quad \sum_{k=1}^{\tilde{N}} \alpha_k = N \ .$$

Here a similar argument as in **Case 1** applies and we get that g has exactly one zero in $\big(-\tilde{b}_{k_0+1}, 1\big)$.

□

Thus we have shown:

Corollary 2.2 (optimal *f* existence)

Let $t_1, \ldots, t_N \in \mathbb{R}$ be a historic trading series, with $\sum_{i=1}^{N} t_i > 0$, and $\hat{t} = \max\Big\{\big|t_i\big| \colon t_i < 0\Big\} > 0$ well defined. Then (4) has a unique solution $f = f^{\,opt} \in (0, 1)$.

3 The drawdown constrained model

In this section we want to discuss the leverage trading model of Vince [6]. Since the notation in [6] is quite unusual, we will introduce the model in our own words. The goal here is to discuss possible drawdowns if we assume that the N historical trades form (1) contribute to a return distribution. We may assume that the trades t_i, $i = 1, \ldots, N$, all have the same probability $1/N$ (Laplace assumption), but that is not necessary.

By drawing randomly $M \in \mathbb{N}$ samples of the return distribution we obtain a probability space

$$\Omega = \{1, \ldots, N\}^M$$

and each random choice $\omega = (\omega_1, \ldots, \omega_M) \in \Omega$ results in a terminal wealth relative

$$\mathrm{TWR}(f, \omega) := \prod_{j=1}^{M} \mathrm{HPR}_{\omega_j}(f) \geq 0, \quad \text{for } f \in [0, 1], \tag{8}$$

of these M trades. Accordingly

$$\mathrm{TWR}_\ell^m(f, \omega) := \prod_{j=\ell}^{m} \mathrm{HPR}_{\omega_j}(f) \geq 0, \; 1 \leq \ell \leq m \leq M, \tag{9}$$

is the relative wealth growth that is obtained using just the $m - \ell + 1$ trades $t_{\omega_\ell}, \ldots, t_{\omega_m}$. In case $\mathrm{TWR}_\ell^m(f, \omega) < 1$ this trade series resulted in a loss and the worst loos that occurs during these M trades given by $\omega = (\omega_1, \ldots, \omega_M)$ is obtained for

$$\min_{1 \leq \ell \leq m \leq M} \min \{\mathrm{TWR}_\ell^m(f, \omega), \; 1\} =: 1 - \mathrm{DD}(f, \omega),$$

where DD stands for (maximal) drawdown. Thus $\mathrm{DD}(f, \omega) \in [0, 1]$. $\mathrm{DD}(f, \omega) = 0$ stands for no loss at all and the larger the drawdown, the larger is the loss. Depending on the individual risk aversion of the investor, one can choose a parameter $c \in (0, 1)$ and whenever a maximal drawdown larger then c occurs, this is considered as ruin. We define the *risk of ruin* (RR) as the probability that such a maximal drawdown $\mathrm{DD}(f, \omega) > c$ occurs, i.e.

$$\mathrm{RR}(f, c) := \sum_{\substack{\omega \in \Omega \\ \mathrm{DD}(f,\omega) > c}} \mathbb{P}(\omega), \tag{10}$$

which in case of a Laplace assumption $\left(\mathbb{P}(\omega) = \frac{1}{N^M}\right)$ simplifies to

$$\mathrm{RR}(f, c) = \frac{1}{N^M} \cdot \sharp \left\{\omega \in \Omega : \mathrm{DD}(f, \omega) > c\right\}. \tag{11}$$

The leverage space trading model of Vince [6] now maximizes the terminal wealth growth $\mathrm{TWR}(f)$ only among those fractions f that guarantee a risk of ruin probability $\mathrm{RR}(f, c) \leq d \in (0, 1)$, where d is again an individual risk aversion parameter of the investor. Typically c and d are small, e.g. $c = 20\%$ and $\mathrm{d} = 1\%$

means that the investor has the restriction that on a trade series of q trades a drawdown that is larger than 20% should occur only with a probability of at most 1%. The resulting optimization problem for $c, d \in (0,1)$ fixed is

$$\begin{aligned} \mathrm{TWR}(f) \;&=\; \prod_{i=1}^{N} \mathrm{HPR}_i(f) \;\stackrel{!}{=}\; \max\,,\;\; f \in [0,1]\,, \\ \text{s.t. } \mathrm{RR}(f,c) \;&\leq\; d\;. \end{aligned} \tag{12}$$

Remark 3.1 *For a large number N of trades and a reasonable M (e.g. $M = 100$) the evaluation of the formula in (10) or (11) is extremely time consuming due to the huge number of elements of Ω. However, replacing Ω by a sufficiently large subset $\tilde{\Omega} \subset \Omega$ that may be determined by random samples gives good approximations of $RR(f,c)$ (cf. [6], pp. 117ff).*

In the remaining we will show that (12) also has a unique solution $f = f_{\mathrm{RR}}^{\mathrm{opt}} \in [0,1]$. To see that, we first need a lemma.

Lemma 3.2 *For fixed $\omega \in \Omega$ and $1 \leq M$ the function*

$$DD(f,\omega) \;=\; 1 - \min_{1 \leq \ell \leq m \leq M} \; \min\left\{ TWR_\ell^m(f,\omega)\,,\; 1 \right\} \tag{13}$$

is continuous and monotonically increasing in $f \in [0,1]$.

Proof: We begin discussing the term $\min\Big\{\mathrm{TWR}_\ell^m(f,\omega)\,,\; 1\Big\}$.

Case 1: All trades t_i, $\ell \leq i \leq m$ are positive or non negative.
Then $\min\Big\{\mathrm{TWR}_\ell^m(f,\omega),\, 1\Big\} \;=\; 1$ is monotonically decreasing in $f \in [0,1]$.
Case 2: There is some $k_0 \in \big\{\ell,\ldots,m\big\}$ such that $\omega_{k_0} = i_0$, i.e. $t_{\omega_{k_0}} = -\hat{t}$.

In this case $\mathrm{TWR}_\ell^m(1,\omega) = 0$ and $\mathrm{TWR}_\ell^m(f,\omega)$ has for $f \in (0,1)$

(A1) exactly one extremum in $(0,1)$ (a maximum with value larger than 1) in case $\sum_{j=\ell}^{m} t_{w_j} > 0$,

(A2) no extremum in $(0,1)$ in case $\sum_{j=\ell}^{m} t_{w_j} \leq 0$.

W.l.o.g., $m > \ell$, because otherwise $m = \ell$ and only **(A2)** with $t_{\omega_\ell} < 0$ has to be discussed. But with $\mathrm{TWR}_\ell^\ell(f,\omega) = \mathrm{HPR}_{\omega_\ell}(f) = 1 - f$ this is obvious.

(A1) now follows immediately from the optimal f lemma, and for **(A2)** obvious adaptions of the proof of this lemma apply (the proof of the claim (7) remains unchanged); see Figure 2.

Case 3: There is no $k_0 \in \big\{\ell,\ldots,m\big\}$ such that $t_{w_{k_0}} = -\hat{t}$, but some $\ell_0 \in \big\{\ell,\ldots,m\big\}$ with $t_{\omega_{\ell_0}} < 0$, and $t_{\omega_{\ell_0}} \leq t_{\omega_j}$, $j = \ell,\ldots,m$

In this case even for $f = 1$ we get that $\mathrm{TWR}_\ell^m\big(f = 1,\omega\big)$ is positive. In fact $\mathrm{TWR}_\ell^m\big(f,\omega\big) > 0$ for all

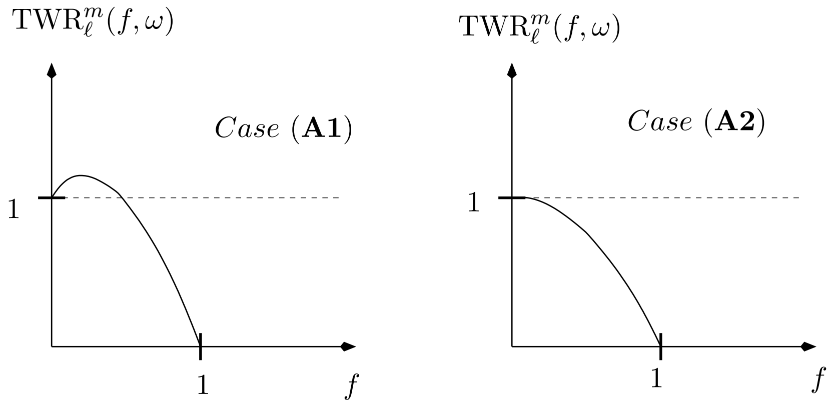

Figure 2: Cases **(A1)** and **(A2)**

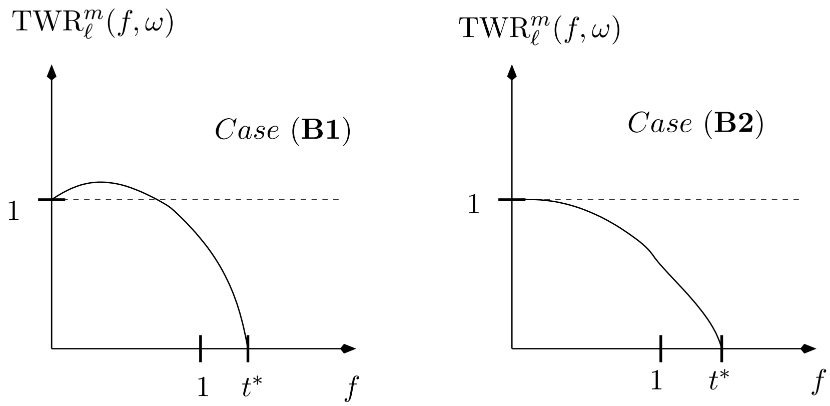

Figure 3: Cases **(B1)** and **(B2)** with $t^* := -\hat{t}/t_{\omega_{\ell_0}} > 1$

$f \in \big[0, -\hat{t}/t_{\omega_{\ell_0}}\big)$. A simple adaption of the arguments from above gives similar situations **(B1)** and **(B2)** which are sketched in Figure 3.
For all of the above cases we get

$$\begin{aligned}&\min\Big\{\mathrm{TWR}_\ell^m(f,\omega),\,1\Big\} \text{ is monotonically decreasing in}\\ &f \in [0,1] \text{ and even strictly monotonically decreasing,}\\ &\text{where } \mathrm{TWR}_\ell^m(f,\omega) < 1\ .\end{aligned}$$

Hence for all fixed $\omega \in \Omega$

$$\min_{1 \le \ell \le m \le M} \ \min\{\mathrm{TWR}_\ell^m(f,\omega)\,,\ 1\}$$

is monotonically decreasing in $f \in [0,1]$, yielding that $\mathrm{DD}(f,\omega)$ is monotonically increasing. The continuity of $\mathrm{DD}(f,\omega)$ is obvious.

□

The immediate consequence of this lemma is that the risk of ruin in (10) is monotonically increasing in f for fixed constant $c \in (0,1)$. Moreover, due to the discreteness of Ω, the risk of ruin $\mathrm{RR}(f,c)$ is piecewise constant and right continuous in f.

Corollary 3.3 (optimal f existence with drawdown constraint)

For fixed $c, d \in (0,1)$ the optimization problem (12) has a unique solution $f = f_{RR}^{opt} \in (0,1)$.

Proof: In case $\mathrm{RR}(f,c) \le d$ for all $f \in [0,1]$ the optimal solution is the one of the optimal f lemma, i.e. $f_{\mathrm{RR}}^{\mathrm{opt}} = f^{\mathrm{opt}}$. Otherwise there exists a unique f^* with

$$\begin{aligned} \mathrm{RR}(f,c) &\le d \quad \text{for all} \quad 0 \le f \le f^* \\ \mathrm{RR}(f,c) &> d \quad \text{for all} \quad f^* < f \le 1 \,. \end{aligned}$$

In this case the unique solution is $f_{\mathrm{RR}}^{\mathrm{opt}} := \min\{f^*, f^{\mathrm{opt}}\}$. □

To finish we will give an application.

Example 3.4 (Kelly betting with CRR 2:1)

A trader has a trading system with a chance–risk–ratio of $2:1$, where the probability to win 2 is $p = 0.4$ and the probability to loose -1 is $q = 1 - p = 0.6$. What is his optimal f if he restricts his position sizing to a maximal drawdown of $c = 10\%$ on $M = 3$ trades with a risk of ruin probability of at most $d = 25\%$?

Answer: *Here we have only two trades $t_1 = -1$ and $t_2 = 2$. With only two outcomes the optimal f ansatz equals the well known Kelly betting system ([2], [1]).*

In this case $f^{opt} = p - q/2 = 10\%$ without any constraint (cf. [5], p. 30). For $M = 3$ trades we have eight possible outcomes $\omega \in \Omega = \Big\{(\omega_1, \omega_2, \omega_3) : \omega_i \in \{1,2\}\Big\}$. In Table 1 we list the maximal drawdowns.

ω	$\mathbb{P}(\omega)$	$\mathrm{DD}(f,\omega)$
$(1,1,1)$	$(0.6)^3$	$1 - (1-f)^3 =: h_3(f)$
$(2,1,1)$ or $(1,1,2)$	$(0.6)^2 \cdot 0.4$	$1 - (1-f)^2 =: h_2(f)$
$(1,2,1)$	$(0.6)^2 \cdot 0.4$	$h_1(f) := \begin{cases} 1-(1-f) = f & \text{for } 0 \le f \le \frac{1}{2} \\ 1-(1-f)^2(1+2f) = 3f^2 - 2f^3 & \text{for } \frac{1}{2} < f \le 1 \end{cases}$
$(1,2,2), (2,1,2)$ or $(2,2,1)$	$0.6 \cdot (0.4)^2$	f
(2,2,2)	$(0.4)^3$	0

Table 1: Maximal drawdowns

For $c \in (0,1)$ *let* $f_i^c \in (0,1)$ *be the unique point with* $h_i\big(f_i^c\big) = c$*. Then* $0 < f_3^c < f_2^c < f_1^c \le c < 1$ *and the risk of ruin is*

$$RR(f,c) = \sum_{\substack{\omega \in \Omega \\ DD(f,\omega) > c}} \mathbb{P}(\omega) = \begin{cases} 0\,, & \text{for} \quad 0 \le f \le f_3^c \\ \big(0.6\big)^3 = 0.216\,, & \text{for} \quad f_3^c < f \le f_2^c \\ \big(0.6\big)^2 \cdot \big(0.6 + 2\cdot 0.4\big) = 0.504\,, & \text{for} \quad f_2^c < f \le f_1^c \\ \big(0.6\big)^2 \cdot \big(0.6 + 3\cdot 0.4\big) = 0.648\,, & \text{for} \quad f_1^c < f \le c \\ 1 - \big(0.4\big)^3 = 0.936\,, & \text{for} \quad c < f \le 1 \end{cases}$$

In our example a risk of ruin of $d = 25\%$ should not be exceeded. For a maximal drawdown of $c = 10\%$, we get $f^* = f_2^c = 1 - 3\sqrt{10}/10 = 5.1\%$. Thus the optimal f for our constraint is $f_{\mathrm{RR}}^{\mathrm{opt}} = \min\Big\{ f^*, f^{\mathrm{opt}} \Big\} = 5.1\%$.

□

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